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Technical Note

The influence of radiation absorption on unsteady free convection and mass transfer flow of a polar fluid in the presence of a uniform magnetic field

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1. Introduction

From the technological point of view, flow arising from temperature and material difference is applied in chemical engineering, geothermal reservoirs, aeronautics and astrophysics. In some applications magnetic forces are present and at other times the flow is further complicated by the presence of radiation absorption, a good example of this is in the planetary atmosphere where there is radiation absorption from nearby stars. Raptis [1] looked at the unsteady free convective flow through a porous medium where it was shown that an increase in the permeability parameter led to an increase in the flow velocity. The interaction of free convection with thermal radiation of an oscillatory flow was studied by Mansour [2] and Stogianidis et al. [3]. Helmy [4] obtained the solution for a magneto-hydrodynamic unsteady free convection flow past a vertical porous plate for a Newtonian fluid and a special type of non-Newtonian fluid known as micro-polar fluids, while Srinivascharya et al. [5] obtained the stream function for the flow and the effects of micro-rotation and frequency parameters on the skin friction of MHD flow between two parallel porous plates.

In all these studies the combined effects of mass and radiative heat transfer in addition to magnetic field have not been considered simultaneously. We now propose to study the effect of radiation heat absorption and mass transfer on the flow of a polar fluid in the presence of a uniform magnetic field since in the astrophysical environment the effect of radiation cannot be neglected.

2. Formulation and analytical solutions

We study the two-dimensional unsteady motion of an incompressible, electrically conducting viscous Boussinesq fluid along an infinite vertical plate in the presence of a uniform magnetic field and radiative heat transfer. In the spirit of Ogulu and Cookey [6], the governing non-dimensional equations proposed here are

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial y^2}-K\frac{\partial^3 u}{\partial t\,\partial y^2}-M(u-1)+G_r\theta+G_c c \tag{1}$$

$$\frac{\partial \theta}{\partial t}=\frac{1}{Pr}\left(\frac{\partial^2}{\partial y^2}-R_a^2\right)\theta+D_f\frac{\partial^2 c}{\partial y^2} \tag{2}$$

$$\frac{\partial c}{\partial t}=\frac{1}{Sc}\frac{\partial^2 c}{\partial y^2} \tag{3}$$

Subject to the boundary conditions

$$\begin{aligned}
&t\leqslant 0\\
&\quad u=1,\ \ \theta=0,\ \ c=0\quad \text{for all } y\\
&t>0\\
&\quad u=1,\ \ \theta=1,\ \ c=1\quad \text{at } y=0\\
&\quad u\to 0,\ \ \theta\to 1,\ \ c\to 1\quad \text{as } y\to\infty
\end{aligned} \tag{4a, b}$$

where u is the velocity, θ is the temperature and c is the concentration along the y-coordinate. The problem as

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formulated in Eqs. (1)–(4) depends on the Schmidt number Sc, the Prandtl number Pr, the free convection parameters due to temperature G_r and concentration G_c respectively, the Hartmann number M (magnetic parameter), the Radiation parameter R, the non-Newtonian parameter K, and the thermo-diffusion term D_f. Assuming an optically thick medium, we invoke the Rosseland approximation for the radiative flux in Eq. (2), as in Raptis [7], neglecting the heat flux in the x-direction, which is assumed small in comparison to that in the y-direction. The mathematical statement of the flow problem now is the solution of Eqs. (1)–(3) subject to the boundary conditions in Eq. (4a,b).

We start the analysis by taking the Laplace transform of Eq. (3) with respect to time, using s as the transformed variable and placing a bar over the transformed function. The result subject to the conditions in (4a,b) can be expressed in the form

$$c(y,t) = \mathrm{erfc}\left\{\frac{y}{2}\sqrt{\frac{Sc}{t}}\right\} \tag{5}$$

Next we take the Laplace transform of Eq. (2) and on appeal to standard Laplace transform tables in Abramowitz and Stegun [8] we can show that

$$\begin{aligned}\theta(y,t) = 0.5&\left[\mathrm{e}^{-y\sqrt{R}}\mathrm{erfc}\left\{\frac{y}{2}\sqrt{\frac{Pr}{t}}-\sqrt{\frac{Rt}{Pr}}\right\}\right.\\ &\left.+\,\mathrm{e}^{y\sqrt{R}}\mathrm{erfc}\left\{\frac{y}{2}\sqrt{\frac{Pr}{t}}+\sqrt{\frac{Rt}{Pr}}\right\}\right]\\ &+0.5PrD_fSc\left[\mathrm{erfc}\left\{\frac{y}{2}\sqrt{\frac{Sc}{t}}-\sqrt{\frac{Rt}{Sc}}\right\}\right.\\ &\left.+\,\mathrm{erfc}\left\{\frac{y}{2}\sqrt{\frac{Sc}{t}}+\sqrt{\frac{Rt}{Sc}}\right\}\right]\end{aligned} \tag{6}$$

First we substitute Eqs. (5) and (6) into Eq. (1), and then we take the Laplace transform of the resultant equation with respect to time. The result is

$$\begin{aligned}\frac{\mathrm{d}^2\bar{u}}{\mathrm{d}y^2}-\beta^2\bar{u} = &-\frac{\mathrm{e}^{-y\sqrt{Pr+R}}}{1-ks}\left(\frac{G_rD_fPrSc}{sSc-Prs-R}+\frac{G_r}{s}\right)\\ &+\frac{\mathrm{e}^{-y\sqrt{sSc}}}{1-ks}\left(\frac{G_rD_fPrSc}{sSc-Prs-R}-\frac{G_c}{s}\right)\end{aligned} \tag{7}$$

where $\beta^2 = \frac{s+M}{1-ks}$. The boundary conditions are

$$\begin{aligned}&\bar{u}(0,s) = \frac{1}{s}\\ &\bar{u}(\infty,s) \to 0\end{aligned} \tag{8}$$

It is difficult to obtain the inverse transform of Eq. (8) for large values of the thermal diffusion parameter, so as in Ogulu et al. [9], we assume the thermal diffusion parameter is small so that on inversion of Eq. (7) subject to the conditions in Eq. (8) we get

$$\begin{aligned}u(y,t) = &(4t)\mathrm{i}^2\mathrm{erfc}\left(\frac{y}{2\sqrt{t}}\right)-\frac{G_c}{kSc}(4t)\mathrm{i}^2\mathrm{erfc}\left(\frac{y}{2\sqrt{t}}\right)\\ &-\left(\frac{Pr+(\sqrt{Pr+Sc})+0.5D_fSc}{Pr-Pr^2+\sqrt{PrSc}(1-Pr)}\right)\mathrm{e}^{Ry}(4t)\mathrm{i}^2\mathrm{erfc}\left\{\frac{y}{2\sqrt{t}}\right\}\\ &+G_r\mathrm{e}^{Ry}(4t)\mathrm{i}^2\left(\frac{Pr+\sqrt{PrSc}+0.5D_fSc}{Pr-Pr^2+\sqrt{PrSc}(1-Pr)}\right)\\ &\times\mathrm{erfc}\left(y\sqrt{\frac{(4Pr+4PrSc+Sc)}{4t}}\right)\\ &+\frac{G_c}{1-Sc}(4t)\mathrm{i}^2\mathrm{erfc}\left\{y\sqrt{\frac{Sc}{4t}}\right\}+\mathrm{e}^{-y\sqrt{M}}\\ &+\mathrm{erfc}\left(\frac{y}{2\sqrt{t}}-\sqrt{Mt}\right)+\frac{G_r}{M}\mathrm{erfc}\left(\frac{y}{2}\sqrt{\frac{Pr}{t}}\right)\\ &+(1-\mathrm{e}^{-Mt})\left[1-\mathrm{erfc}\left(\frac{y}{2\sqrt{t}}\right)\right]\end{aligned} \tag{9}$$

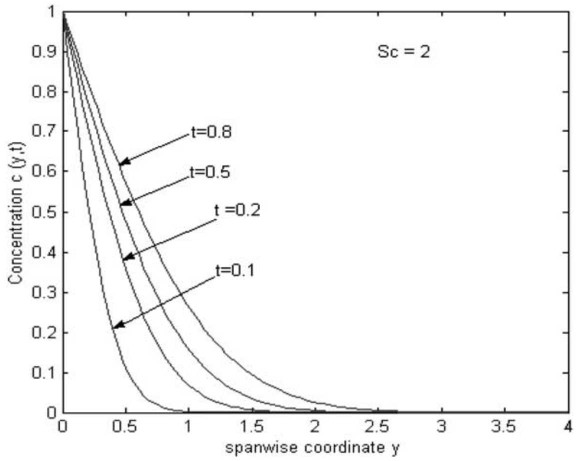

Fig. 1. Concentration distribution along span-wise coordinate.

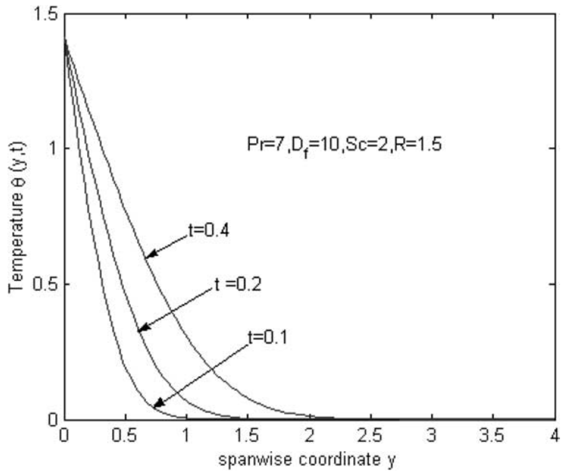

Fig. 2. Temperature distribution along span-wise coordinate.

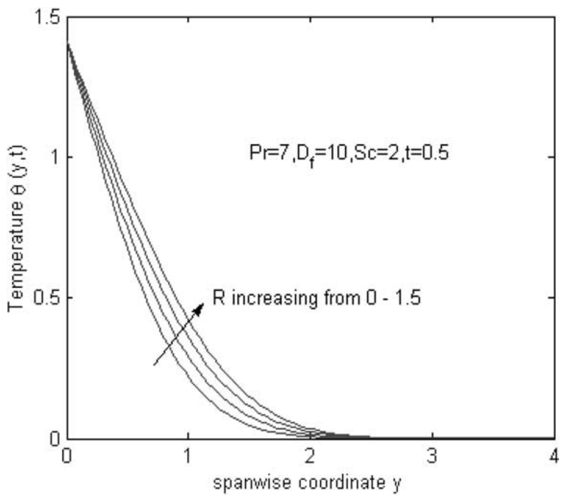

Fig. 3. Effect of radiation parameter on the temperature distribution.

3. Discussion and conclusion

We observe from Figs. 1 and 2 that both the concentration and the temperature distributions increase with time, but generally decrease exponentially with increase in span-wise distance. This trend is more for the concentration than for the temperature, and also for lower values of t than for higher values. When other parameters are held constant, it is observed that the temperature increases with increase in the radiation parameter, Fig. 3. This observation is in agreement with the earlier reported work of Cookey et al. [10]. Indeed as seen from Eq. (1) the velocity is coupled to the temperature and the concentration via the free convection parameters so that changes in these parameters have a direct effect on the velocity.

In conclusion, the study has shown among other things that the inclusion of heat absorption due to radiation in the problem is of particular relevance especially in astrophysical studies and as such would be of interest in the re-entry problem.

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